Cohn's criterion is based on the behavior of the fringing capacities for $t=0$ and one might expect intuitively that the interaction of fringing capacities would decrease markedly as t increases.

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Coupled Power Equations for Backward Waves

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***Abstract*—Two waves traveling in opposite directions that are coupled by a random coupling function are considered. These two waves can be described in a standard way by coupled wave equations. It is possible to derive coupled equations for the power carried by these two waves. The form of the coupled power equations differs depending on the assumptions that are made for the initial conditions. The validity of the coupled power equations has been confirmed by a computer-simulated experiment.**

Introduction

COUPLED POWER equations for waves traveling in opposite directions have been derived by Rowe [1] under the assumption that the coupling function has a white-noise spectrum and that the initial conditions for both waves have been specified at the far end of the transmission lines. He thus assumes that the output of mode (or line) 1 is specified at the end of the guide and that no power is incident at the far end in the reflected wave. His theory predicts the expected value of the reflected wave at the input of the line, as well as the expected values of the input waves that are required to obtain the fixed output value of the incident mode.

If one considered it as an established fact that the power exchange between the two waves can be treated by adding power instead of amplitude, one would write down intuitive coupled power equations that differ in form from the coupled power equations that Rowe derived. The question arises whether those intuitive equations are meaningless or how they are related to Rowe's equations. In order to gain insight into that problem, we conducted a computer-simulated experiment tracing waves through ten simulated waveguides with random coupling and compared the average output power obtained from the experiment with the prediction of the theories. The experiment can be done in several ways. It is possible to launch a constant amplitude into each of the ten random waveguides and to compute the average values of the power output of the incident wave at the far end of the guide, as well as the average power of the reflected wave at the near end of the guide. The result of this experiment agreed strikingly with the intuitive coupled power equations, while it was definitely at odds with Rowe's equations. However, the experimental conditions did not conform to Rowe's assumptions. We then changed the conditions requiring that the output voltage of the incident wave have a fixed value at the far end while no power enters the reflected mode at the far end. The experimental values now showed far larger scatter than in the first case, but comparison indicated that they were in agreement with Rowe's equations while they definitely contradicted the predictions of the intuitive equations if they were applied to this case.

The result of this experiment points to the conclusion that different differential equations are required to describe the statistical outcome of coupled wave experiments in which the two waves travel in opposite directions. One set of equations describes the situation in which the input wave is known while no reflected wave is allowed to enter at the far end. Another set of coupled power equations describes the experimental situation in which we require that the output wave has a definite amplitude, while again no power is allowed to enter the reflected mode at the far end.

In this paper both types of coupled power equations are derived from the coupled wave equations using per-

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turbation theory. The derivation of these equations is not rigorous, and no attempt has been made to verify the assumptions that are needed for their derivation. However, one set of equations is in agreement with Rowe's rigorous theory and the computer-simulated experiment aids in establishing confidence that the other set of equations derived in this nonrigorous way is indeed valid.

Derivation of Coupled Power Equations

Coupled power equations have been derived from coupled wave equations by several authors [2], [3]. A derivation of coupled power equations for N modes ($N \geq 2$) by the same method employed in this paper has been published earlier [4].

Our starting point will be the coupled wave equations [5] for two modes traveling in opposite directions:

$$\frac{da}{dz} = -i\beta_1 a + c_{12} b \tag{1}$$

$$\frac{db}{dz} = i\beta_2 b + c_{21} a. \tag{2}$$

We assume that the wave with amplitude a travels in the positive z direction with propagation constant β_1, while the wave with amplitude b travels in the negative z direction with propagation constant β_2. We shall assume that β_1 and β_2 are both real neglecting losses in the waveguide. The substitutions

$$a(z) = A(z)e^{-i\beta_1 z} \tag{3}$$

$$b(z) = B(z)e^{i\beta_2 z} \tag{4}$$

introduce the slowly varying wave amplitudes A and B. In fact, in the absence of coupling we would have A = constant and B = constant. Substitution of (3) and (4) into (1) and (2) results in the reduced form of the coupled wave equations:

$$\frac{dA}{dz} = c_{12} B e^{i(\beta_1+\beta_2)z} \tag{5}$$

$$\frac{dB}{dz} = c_{21} A e^{-i(\beta_1+\beta_2)z}. \tag{6}$$

Conservation of power requires the relation

$$\frac{d|A|^2}{dz} - \frac{d|B|^2}{dz} = 0. \tag{7}$$

The minus sign is required since the amplitude B belongs to a wave traveling in the negative z direction. When the wave A gains power its z derivative is positive. However, if the wave B gains power it grows as it travels along the negative z axis so that its z derivative is negative. The sum of the power gain of the two waves must vanish if power is to be conserved. Using the differential equations (5) and (6) we obtain from (7)

$$(c_{12} - c_{21}^*)A^*Be^{i(\beta_1+\beta_2)z} + \text{c.c.} = 0.$$

The asterisk indicates complex conjugation. The abbreviation c.c. indicates that the complex conjugate of the terms appearing in the equation must be added. Because A and B can be chosen arbitrarily owing to our freedom of choice of initial conditions, we can satisfy this equation only by requiring the following relation between the coupling coefficients:

$$c_{12} = c_{21}^*. \tag{8}$$

Since our objective is to derive coupled equations for the average power carried by the two modes, we form

$$\frac{d\langle |A|^2\rangle}{dz} = \langle c_{12}A^*B\rangle e^{i(\beta_1+\beta_2)z} + \text{c.c.} \tag{9}$$

where $\langle\ \rangle$ indicates an ensemble average.

In order to be able to use perturbation theory we require that the coupling is sufficiently weak so that the wave amplitudes A and B change only very little over distances comparable to or larger than the correlation length D of the correlation function

$$R(u) = R(-u) = \langle c_{12}(z)c_{12}^*(z+u)\rangle. \tag{10}$$

The idea of our derivation is based on the intuitive expectation that the wave amplitudes at a point $z = z'$ are uncorrelated with the coupling coefficients at a point $z = z' \pm \Delta z$ with $\Delta z \gg D$. In order to evaluate (9) we proceed in two different ways. First, we use the following perturbation solution of (5) and (6):[1]

$$A(z) = A(z' - \Delta z) + B(z' + \Delta z) \cdot \int_{z'-\Delta z}^{z} c_{12}(x)e^{i(\beta_1+\beta_2)x}\,dx \tag{11}$$

$$B(z) = B(z' + \Delta z) + A(z' - \Delta z) \cdot \int_{z'+\Delta z}^{z} c_{12}^*(x)e^{-i(\beta_1+\beta_2)x}\,dx. \tag{12}$$

It should be noted that the two equations are based on different assumptions. In (11) we assumed that the amplitude A is known at a point $z' - \Delta z$ to the left of the point z. The argument $z' + \Delta z$ of B appearing in (11) was chosen since B is traveling in the backward direction so that it is natural to assume that we know the value of B at the point $z' + \Delta z$. The fact that B has been taken out of the integration sign is in keeping with our perturbation assumption that neither A nor B vary very much over the distance Δz.

In (12) we expanded $B(z)$ backwards assuming that B

[1] The coordinate z is the variable of the differential equations (5) and (6), while z' is used as a constant reference point in the vicinity of z.

is known at $z'+\Delta z$. The coupling coefficient c_{21} was replaced with c_{12}^* with the help of (8).

We now substitute (11) and (12) into (9) and neglect terms of order higher than the second in the coupling coefficient c_{12}. We thus obtain

$$\frac{d\langle |A(z)|^2\rangle}{dz} = \langle |A(z'-\Delta z)|^2\rangle \cdot \int_{z'+\Delta z}^{z} \langle c_{12}(z)c_{12}^*(x)\rangle e^{i(\beta_1+\beta_2)(z-x)}\,dx + \langle |B(z'+\Delta z)|^2\rangle \cdot \int_{z'-\Delta z}^{z} \langle c_{12}(z)c_{12}^*(x)\rangle e^{i(\beta_1+\beta_2)(z-x)}\,dx + \text{c.c.} \tag{13}$$

We used our assumption that A and B, at a point $z'\pm\Delta z$, are uncorrelated with c_{12} at z'. This assumption permitted us to write ensemble averages of products of amplitudes and coupling coefficients as products of ensemble averages of amplitudes times ensemble averages of coupling coefficients. The term linear in c_{12} vanished because we assume

$$\langle c_{12}\rangle = 0. \tag{14}$$

We rewrite the integrals appearing in (13) in the following way:

$$\int_{z'+\Delta z}^{z} \langle c_{12}(z)c_{12}^*(x)\rangle e^{i(\beta_1+\beta_2)(z-x)}\,dx = -\int_{-\infty}^{0} R(u)e^{i(\beta_1+\beta_2)u}\,du \tag{15}$$

and

$$\int_{z'-\Delta z}^{z} \langle c_{12}(z)c_{12}^*(x)\rangle e^{i(\beta_1+\beta_2)(z-x)}\,dx = \int_{-\infty}^{0} R(u)e^{-i(\beta_1+\beta_2)u}\,du. \tag{16}$$

We used (10) and replaced the lower integration limit with $-\infty$, assuming that the correlation function decreases so rapidly for $u>D$ that the change of the integration limit has no influence on the value of the integral. We introduce the abbreviations

$$P_a(z') = \langle |A(z')|^2\rangle \approx \langle |A(z'-\Delta z)|^2\rangle \tag{17}$$

$$P_b(z') = \langle |B(z')|^2\rangle \approx \langle |B(z'+\Delta z)|^2\rangle \tag{18}$$

and

$$K = 2\int_{-\infty}^{0} R(u)\cos(\beta_1+\beta_2)u\,du \tag{19}$$

and obtain from (7) and (13) the following set of coupled power equations:

$$\frac{dP_a}{dz} = -K(P_a - P_b) \tag{20}$$

$$\frac{dP_b}{dz} = -K(P_a - P_b). \tag{21}$$

These equations could have been written down immediately since they have a simple intuitive meaning. Assume that $P_b=0$. We then see from (20) that P_a decreases since it loses power to mode B. Similarly we obtain a positive derivative for P_b from (21) by assuming $P_a=0$. This too indicates that power is lost from mode B to mode A since B is a mode traveling in the negative z direction. Finally, it appears plausible to require vanishing derivatives if $P_a=P_b$.

For our second derivation of coupled power equations we assume that A is not given at a point to the left of $z=z'$, but use instead the assumption that A is known at a larger z value and write, in complete analogy with (12),

$$A(z) = A(z'+\Delta z) + B(z'+\Delta z) \cdot \int_{z'+\Delta z}^{z} c_{12}(x)e^{i(\beta_1+\beta_2)x}\,dx. \tag{22}$$

It is apparent that we now must use, instead of (12),

$$B(z) = B(z'+\Delta z) + A(z'+\Delta z) \cdot \int_{z'+\Delta z}^{z} c_{12}^*(x)e^{-i(\beta_1+\beta_2)x}\,dx. \tag{23}$$

Proceeding in exact analogy to the derivation of (20) and (21) we obtain the following set of coupled power equations:

$$\frac{dP_a}{dz} = -K(P_a + P_b) \tag{24}$$

$$\frac{dP_b}{dz} = -K(P_a + P_b). \tag{25}$$

Equations (24) and (25) do not have a simple intuitive meaning and in fact appear wrong on the basis of the argument brought forth to explain the meaning of (20) and (21). However, (24) and (25) can be obtained from Rowe's paper [1] in the limit of vanishing losses and weak coupling. Our derivation suggests that (20) and (21) hold when we specify that A is known for values of z smaller than the point at which we wish to apply the differential equations, while B is known at a larger value of z. Equations (24) and (25) were derived under the assumption that both A and B are specified for z values larger than the value at which the differential equation is to be applied. It is thus not surprising that the set of equations (20) and (21) can be used to solve the random coupling problem if we use the boundary conditions $A(0)=1$ and $B(L)=0$. Equations (24) and (25), on the other hand, apply to the case considered by Rowe: $A(L)=1$ and $B(L)=0$.

A third form for the coupled power equations can be obtained by assuming that both A and B are known for z values smaller than the value at which the differential equations are to be applied. These equations differ from (24) and (25) by the fact that both equations now appear with a plus sign.

Computer-Simulated Experiment

In order to check the validity of the coupled power equations, we conducted a computer-simulated experiment. In order to use a simple model for the random coupling, we assumed that c_{12} has a constant magnitude, but randomly varying sign:

$$c_{12} = \pm\kappa. \tag{26}$$

The sections over which c_{12} remains unchanged are given constant length D. The correlation function is then given by

$$R(u) = \begin{cases} \kappa^2 \dfrac{D - |u|}{D}, & \text{for } |u| \leq D \\ 0, & \text{for } |u| > d \end{cases} \tag{27}$$

while the Fourier transform of the correlation function is

$$K = \frac{\kappa^2}{2D\beta^2} (1 - \cos 2\beta D). \tag{28}$$

For simplicity $\beta_1 = \beta_2 = \beta$ was assumed.

The coupled power equations (20) and (21) have the following solutions:

$$P_a = P_0 \frac{1 + K(L - z)}{1 + KL} \tag{29}$$

and

$$P_b = P_0 \frac{K(L - z)}{1 + KL}. \tag{30}$$

Built into these solutions are the boundary conditions $P_a(z) = P_0$ at $z = 0$ and $P_b(z) = 0$ at $z = L$. In order to check the validity of the solutions (29) and (30) we have computed the matrix relating the amplitudes of the output of a transmission line by multiplying all the matrices belonging to the individual sections of length D. Each of these matrices has the form

$$M_i = \begin{vmatrix} \cos \beta' D - i \dfrac{\beta}{\beta'} \sin \beta' D & \pm \dfrac{\kappa}{\beta'} \sin \beta' D \\ \pm \dfrac{\kappa}{\beta'} \sin \beta' D & \cos \beta' D + i \dfrac{\beta}{\beta'} \sin \beta' D \end{vmatrix} \tag{31}$$

with

TABLE I

$2\beta D = \pi/2$, 100 sections of guide with constant coupling of random sign

$2\pi\kappa/\beta$	$P_a(L)$ (exp)	$P_a(L)$ (theo)	$P_b(0)$ (exp)	$P_b(0)$ (theo)
0.1	0.990	0.988	0.00999	0.0124
0.3	0.913	0.899	0.0877	0.101
1.0	0.433	0.444	0.567	0.556
3.0	$9.8\ 10^{-5}$	$8.1\ 10^{-2}$	0.999	0.919
500 sections of guide with constant coupling of random sign				
0.1	0.949	0.940	0.0510	0.0595
0.3	0.633	0.637	0.367	0.363
1.0	0.0496	0.137	0.950	0.863

$$\beta' = \sqrt{\beta^2 - \kappa^2}. \tag{32}$$

The matrix relating input amplitudes to output amplitudes for the entire length of waveguide is

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = M_N \cdot M_{N-1} \cdots M_2 \cdot M_1. \tag{33}$$

The amplitudes at the beginning and end of the guide are related by the equation

$$\begin{pmatrix} a_N \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ b_1 \end{pmatrix}. \tag{34}$$

Generating random sequences of $c_{12} = +\kappa$ and $c_{12} = -\kappa$ allows us to simulate random waveguides. The average output power at the end of these guides is $P_a(L) = |a_N|^2$ and the average reflected power arriving at $z = 0$ is $P_b(0) = |b_1|^2$. For simplicity we used $P_0 = 1$.

The result of the simulated experiment is compared with theory in Table I. Each experimental average value is the result of using waveguides with 100 (or 500) sections with constant coupling coefficients, but with randomly varying signs and averaging over 10 such random waveguides. The dependence of the average power on $2\beta D$ via (28)–(30) has been checked and was found to be in excellent agreement with experiment. Table II shows two samples of the individual results obtained for all 10 simulated waveguides, and gives some idea of the fluctuations of the actual power around the average value.

A similar comparison was made by considering, instead of (34), the boundary conditions

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \tag{35}$$

and comparing the experimentally obtained power averages with the solutions of (24) and (25):

TABLE II

$2\beta D = \pi/2$, 100 sections of guide with constant coupling of random sign

$2\pi\kappa/\beta = 0.3$

number of experiment	$P_a(L)$ (exp)	$P_b(0)$ (exp)
1	0.8635	0.1365
2	0.8538	0.1462
3	0.8849	0.1151
4	0.8920	0.1080
5	0.9488	0.0512
6	0.9299	0.0701
7	0.9793	0.0207
8	0.9078	0.0922
9	0.9586	0.0414
10	0.9137	0.0863
	$\langle (P_b - \langle P_b \rangle)^2 \rangle^{1/2} = 0.0416$	
500 sections of guide with constant coupling of random sign		
1	0.5413	0.4587
2	0.5738	0.4262
3	0.4021	0.5979
4	0.6503	0.3497
5	0.9002	0.0998
6	0.5171	0.4829
7	0.8372	0.1628
8	0.7926	0.2074
9	0.7111	0.2889
10	0.4038	0.5962
	$\langle (P_b - \langle P_b \rangle)^2 \rangle^{1/2} = 0.166$	

$$P_a = \tfrac{1}{2}(e^{2KL} + 1) \qquad (36)$$

$$P_b = \tfrac{1}{2}(e^{2KL} - 1). \qquad (37)$$

The agreement of the experimentally obtained averages with the theoretical results was much poorer in this case. The poor agreement can be attributed to the very large scatter in the data of the simulated experiment. However, the experimental rseults were definitely in favor of the theory (36) and (37) and did not agree at all with the solutions (29) and (30) of the power equations (20) and (21). It is thus apparent that the two different sets of differential equations (20) and (21) on the one hand and (24) and (25) on the other hand are required to describe the two experimental situations. The statistical differential equations thus have the unusual property of being directly connected with the boundary conditions that are to be imposed on the solution. Ordinarily, a differential equation (or a set of equations) is given independently of the boundary conditions. Its solutions are selected by the requirement that they satisfy certain boundary conditions. Our statistical equations have the feature that we need the boundary conditions not only to select the proper solutions of the equations but also to select the proper differential equations that are compatible with just these boundary conditions.

In order to explain the seeming anomaly we consider the following simple example [6]. Let us assume that two variables x and y are related by the equation

$$x = ay. \qquad (38)$$

If a is a random variable we can calculate the expected values of x provided y is specified. We thus obtain

$$\langle x \rangle = \langle a \rangle y. \qquad (39)$$

On the other hand we can also consider x as specified and calculate the expected value of y:

$$\langle y \rangle = \left\langle \frac{1}{a} \right\rangle x. \qquad (40)$$

Since the expected values of a and $1/a$ are different from each other, the functional relationship (39) is not the same as (40). The problem is inherently nonlinear so that the result depends on which of the variables is considered as specified and which is considered as random. This is just the distinction between the different types of initial value problems that we have considered in this paper.

Conclusions

We have found that two modes traveling in opposite directions in a waveguide with random coupling coefficients can be described by coupled power equations describing the interchange of power from one mode to the other on a statistical (ensemble average) basis. Using perturbation theory it is possible to derive different forms of these coupled power equations. Each form is valid in conjunction with a certain set of boundary conditions. We thus have the unusual situation that the boundary conditions determine not only the particular solutions of a given differential equation but that the differential equation itself is related to the boundary conditions. The derivation of the coupled power equations is mathematically not rigorous, but is based on ideas of perturbation theory. The validity of the equations has been confirmed by a computer-simulated experiment that was performed for a particularily simple model of the random coupling function. The agreement of the computer simulation with one set of differential equations is excellent. The other set of differential equations describes conditions for which the experimental data resulted in a great deal of scatter. However, this set of coupled power equations has independently been derived by a rigorous method by Rowe so that no need was felt to improve the agreement between theory and experiment by using larger statistical samples, since the experimental evidence clearly suggested the validity of these equations.

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Short Papers

On Optimum Mirrors of the Fabry–Perot Resonator Filled with Anisotropic Medium

TOSHIKI TANAKA AND MICHIO SUZUKI

***Abstract*—A Fabry–Perot resonator formed by parabolic cylindrical reflecting mirrors and filled with anisotropic medium is analyzed theoretically. For the resonance of the extraordinary wave in such a resonator, optimum shapes of the reflecting mirrors exist, which are derived from the boundary conditions on each mirror.**

Fabry–Perot resonators (FPRs) are widely used in the optical frequency region, and the confocal FPR is known as a resonator with considerably low diffraction loss [1], [2]. However, where the medium in the resonator is anisotropic and the resonance of the extraordinary wave, which has E_y, E_z, and H_x components as shown in Fig. 1 is required, the diffraction loss increases greatly. This effect depends on the fact that the reflections of the extraordinary wave at the mirrors' surfaces are not symmetrical because of the anisotropy.

In this short paper we present the theoretical results on the optimum shapes of the reflecting mirrors for the resonance of the extraordinary wave.

We consider an FPR formed by two parabolic cylindrical reflecting mirrors and filled with anisotropic medium as shown in Fig. 1. The dyadic dielectric constant of the medium is given by

$$\bar{\epsilon} = \sum_{i=1}^{3}\sum_{j=1}^{3} n_i n_j \epsilon_{ij} \tag{1}$$

where we assume that $\epsilon_{12} = \epsilon_{21} = \epsilon_{13} = \epsilon_{31} = 0$.

When a current distribution $J_y(y, z)$ exists in the resonator, the total electric field in the resonator $E_t(y, z)$ is given by the sum of $E_s(y, z)$, due to $J_y(y, z)$, and $E_m(y, z)$, due to the induced mirror currents $J_1(y, z)$ and $J_2(y, z)$. Assuming that the surface impedances of the mirrors are Z_{S_1} and Z_{S_2}, and taking $a \ll b$, the boundary conditions of the electric field on S_1 and S_2 are as follows:

$$\begin{aligned} E_{m,y}(S_1) + E_{s,y}(S_1) - Z_{S_1} J_1(y) &= 0 \\ E_{m,y}(S_2) + E_{s,y}(S_2) - Z_{S_2} J_2(y) &= 0. \end{aligned} \tag{2}$$

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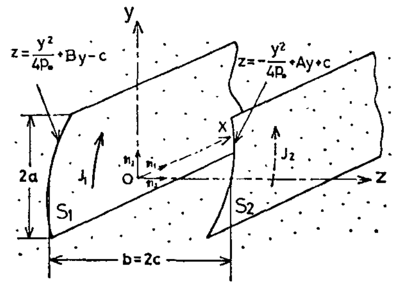

Fig. 1. Fabry–Perot resonator filled with anisotropic medium.

Assuming that the equations of the mirrors S_1 and S_2 are

$$\begin{aligned} Z = g_1(y) &= \frac{1}{4p_0} y^2 + By - C \\ Z = g_2(y) &= -\frac{1}{4p_0} y^2 + Ay + C \end{aligned} \tag{3}$$

we can calculate the electric field $E_{m,y}$ and $E_{s,y}$ by employing the uniform transmission line representation [3], regarding the direction z as a transmission line. Substituting the results into (2) and changing the variable $y = at$, we obtain the following simultaneous integral equations:

$$\begin{aligned} \left(1 + \frac{2Z_{S_1}}{R_0}\right) J_1(t) + \int_{-1}^{1} J_2(t') K_{21}(t, t') dt' &= \epsilon_{s,y}(S_1) \\ \int_{-1}^{1} J_1(t') K_{12}(t, t') dt' + \left(1 + \frac{2Z_{S_2}}{R_0}\right) J_2(t) &= \epsilon_{s,y}(S_2) \end{aligned} \tag{4}$$

where

$$\begin{aligned} K_{21}(t, t') &= \sqrt{\frac{c_1}{2\pi}} \exp -j\left[kb_a - \frac{\pi}{4} + \frac{c_0}{2}(t^2 + t'^2) \right. \\ &\qquad \left. - c_1 tt' + c_2' t - c_2 t' \right] \\ K_{12}(t, t') &= \sqrt{\frac{c_1}{2\pi}} \exp -j\left[kb_a - \frac{\pi}{4} + \frac{c_0}{2}(t^2 + t'^2) \right. \\ &\qquad \left. - c_1 tt' + c_2 t - c_2' t' \right] \end{aligned} \tag{5}$$